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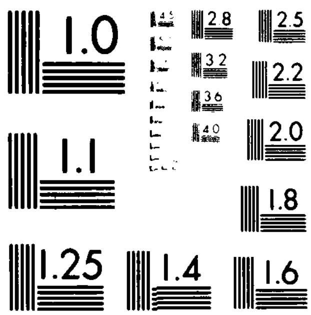

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A-7 FLIGHT SOFTWARE ANALYSIS

Bruce B. Amlicke
Joseph C. Fox

February 1982

Prepared for: Naval Research Laboratory
455 Overlook Avenue, S.W.
Washington, D.C. 20375

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This document describes the development of Unitless Mathematical Models for on-board flight software. The development is part of the NRL Software Cost Reduction Program. A subset of the equations existing in the operational flight program were examined to determine existing units and basic assumptions. The equations were then re-derived in unitless form with clearly stated assumptions; this is the form which is most useful to modular design.

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TABLE OF CONTENTS

Section		Page
ACKNOWLEDGEMENTS		i
ABSTRACT		ii
1	Introduction	1-1
	1.1 Background	1-1
	1.2 Techniques for Generating Unitless Models	1-2
	1.3 Report Organization	1-4
2	Air Data Computer	2-1
	2.1 General	2-1
	2.2 Altitude Correction for Non-AIMS-probe and AIMS-probe Systems	2-1
	2.3 Correction to Altitude for Non-Standard Sea Level Pressure	2-14
	2.4 References	2-25
3	Angle of Attack (AOA)	3-1
	3.1 General	3-1
	3.2 AOA Scaling and Smoothing	3-1
	3.3 References	3-10
4	Doppler Radar Set	4-1
	4.1 General	4-1
	4.2 Corrections to Doppler Ground Speed	4-1
	4.3 References	4-7
BIBLIOGRAPHY		Bib-1

Appendix		
A	List of Symbols	A-1
B	Real Variable Types	B-1
C	Dependencies of Constants used in Physical and Empirical Equations	C-1
D	Notation	D-1

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ABSTRACT

This document describes the development of Unitless Mathematical Models for on-board flight software. The development is part of the NRL Software Cost Reduction Program. A subset of the equations existing in the operational flight program were examined to determine existing units and basic assumptions. The equations were then re-derived in unitless form with clearly stated assumptions; this is the form which is most useful to modular design.

SECTION 1
INTRODUCTION

1.1 Background

The concepts of Modular Mathematics and Information Hiding are now being applied to software for the A-7 aircraft on-board computer. This demonstration program has been active for several years. However, to fully apply these concepts, it has become evident that a detailed analysis of major portions of the basic mathematics is required. There are several reasons for this. First, there are a number of equations within the Operational Flight Program (OFP) that contain constants with units which are unknown or do not conform to the basic data types. It is not evident by inspection how these constants were determined or on what they might depend. That is, it is not apparent how such constants depend on basic quantities with known physical meanings. Second, the mathematics also depend on certain basic assumptions not readily discernible in the form in which the equations are presented. The nature of these assumptions must be known so that a given model may be placed into a specific module. Then, if a basic assumption changes, the software modification should be confined to a single module. Finally, the equations used in the OFP are not in a form which supports modularity. Instead, the equations produce intermediate products which do not conform to the established basic data types.

Modular Mathematics can be usefully defined as, "the statement of the governing equations in terms of known assumptions using 'universal' constants and in a unitless formulation." Modular Mathematics supports the principles of Information Hiding by allowing the user to be concerned only with constants of known physical meaning and not with the actual units used.

Aside from being a necessary condition for the employment of the principle of Information Hiding, Modular Mathematics is useful in its own right. It provides a cost reduction during the development phase by highlighting assumptions inherent in the mathematical statement of the problem. Thus, an algorithm that optimizes the time, accuracy, and storage requirements can be selected. A carefully compiled set of equations which are rigorously derived and fully documented should be highly reusable. For example, a well-written and documented subprogram will find frequent reuse to meet similar needs in other systems. It will also serve as a model and point of departure for improved versions of the same subprogram or other subprograms developed to meet related tasks. Thus, a well-supported mathematical development will serve the user who only needs the mathematics to solve a similar problem or needs a good example to follow in other mathematical developments, in perhaps unrelated areas. Once a set of software is developed, the modular form of the mathematics can provide substantial aid during program maintenance. A change in device or system specification can be relatively quickly traced to the relevant equations and from there to the point requiring modification in the derivation or change in the constants. Then the change can be made rapidly and with little danger of any widespread, undesired effect. Substantial cost savings can be realized by not having to reproduce the mathematics from first principles for each change in device or system specification.

1.2 Techniques for Generating Unitless Models

A Unitless Model is not only a mathematical prescription which is free of unknown constants or operations with inappropriate data types; it is the entire derivation behind that equation, including the origin of the relevant equation and the sequence of mathematical operations that follow to arrive at the final useful form. The trail includes all assumptions made and all constants identified and introduced. In an ideal situation, the model could be constructed from scratch. Consideration could be given to such issues as accuracy, machine time required, storage required, error propagation, and simplicity of form. In many cases, the mathematics already exists,

so the origin and derivation must be determined from the form of the equation and knowledge of the relevant processes.

The mathematical analysis is approached by considering a detailed statement of the problem taken from the description of the relevant module. The module description may not contain enough detailed information to set up entirely the mathematical problem, and the description must, therefore, be expanded to include a sequence of steps or operations that will accomplish this task. The new problem statement must be examined to determine if it still adheres to the basic tenents of the module and if it is a clear and concise statement. In many cases, several derivations are possible. The selection between them may be performed at this point in the development if there is a clear, compelling, distinction between them. If the decision is not made here, then the alternative forms may be carried forward for a decision at a later point in the development, based on additional information.

The first step is to select a starting point for the mathematical development. It should be as general and free from restrictive assumptions as possible. Consider a ballistics problem, for example. Rather then stating that a force is equal to mass times acceleration, we could start with the more general case of conservation of momentum. We would then argue that since mass does not change, the result, F=ma, is applicable. By starting from the more general statement, we have highlighted an assumption (i.e., that mass does not change) which is implicit to the problem. Thus, in selecting a point of origin, one should use the most general statement possible. Of course, this principle can be carried to extreme. Thus, that point of origin should be selected that includes the full range of different assumptions encountered, yet takes advantage of assumptions that are never expected to change.

The mathematical development should be as mathematically rigorous as is consistent with the manner in which the equations are used. The assumptions and approximations made during the development must be justifiable and

consistent with usage of the final form. These assumptions should be carefully noted in the development. In the case where there is no preexisting final form of the equation, consideration must be given to required accuracy, storage, timing, and other parameters in selecting the end point of the development. Accuracy considerations require that the equations derived include all corrections which have a measurable effect somewhere in the module result. However, effects several orders of magnitude down from the accuracy level of the given quantity should be ignored. Since no computer offers truly unlimited resources of time and storage, consideration must be given to minimizing the use of both. The traditional programming technique to reduce computational time is to store as many constants as possible; on the other hand, to minimize storage, constants are typically recalculated. In the context of Mathematical Modularity, either approach must be used with discretion. Constants selected for storage must have a recognizable physical meaning so that they can be available to the correct module. Further, constants should have a dimensionality consistent with the basic data types.

Once the final equation is derived, it must be posed in a form which is consistent with the basic data types. The basic data types are selected to meet the needs of the problem at hand. They are the minimum set which meet the needs of the program while not taxing the system. The final form of the equation should include only those operations which produce one of the basic data types as the end product.

1.3 Report Organization

This report documents the results of the initial phase of the project to generate Unitless Models for the A-7 flight software. The report is divided into chapters which parallel the division of the program into modules. Each chapter is self-contained and includes lists of symbols and references pertaining to that chapter; more chapters can be added as

additional modules are analyzed. This will also allow corrections to be made to a single chapter without the necessity of changing anything in any other chapter. The notation and references used are consistent throughout the text and are also listed separately in appendixes.

SECTION 2
AIR DATA COMPUTER

2.1 General

The Air Data Computer (ADC) processes inputs from the Pitot-static system and the total temperature probe to provide outputs that represent barometric altitude, Mach number, and true airspeed.

2.2 Altitude Correction for Non-AIMS-probe and AIMS-probe systems

Background

Page ADC-13 of Reference 1 on the ADC gives two equations,

```
alt := alt + [560. *(cmindex(mi) - .2)]                         (2-1)
```

and

```
alt := alt * {1. - cmindex(mi)**2
       *[.02032 - ((7.5148E-7)*alt)]}
       +CONVERT(!system vertical velocity!).                    (2-2)
```

These equations correct the measured value of the barometric altitude for errors associated with the location of the Pitot-static port.

There are two Pitot-static systems available for the A-7 aircraft. The first has the static ports along the side of the aircraft just below the canopy rail. The second has its static ports located on the Pitot tube, which is an L-shaped tube protruding from the side of the aircraft just aft of the radome. The second system is referred to as an AIMS (or in some cases an L-probe) system, and the first as a non-AIMS system (or non-L-probe system). Equation 2-1 provides correction on the non-AIMS-probe aircraft, while Equation 2-2 refers to AIMS aircraft. Several quantities need to be defined.

Static Pressure - The absolute pressure of the still air surrounding the aircraft.

Static Defect - The difference between the pressure at various places on the aircraft skin and the free-stream pressure.

Position Error - The error in the static pressure at the static port due to static defect.

The pressure on the aircraft skin may be either slightly lower or higher than the free-stream pressure. The static defect at a particular location depends on speed, angle of attack, and altitude. To obtain a sample of static air on a moving aircraft, a hole (static port) or a series of holes can be drilled in a plate on the side of the fuselage or on the side of a rod-like probe (called a Pitot-static tube) extending into the free-air stream. The location of the static port is selected by wind-tunnel tests and by tests at numerous locations on the actual aircraft. The static port is connected to a pressure transducer by means of tubing. The time constant of the cavity formed by the tubing and the transducer is a function of the viscosity of the air, cavity volume, static pressure, and length and diameter of the connecting tubing.

The time required for a change in static pressure to be sensed by the Pitot-static system on a maneuvering aircraft introduces further errors. An additional term must be defined.

Pressure Lag Error - The time delay between the change in static pressure and the change in signal from the pressure transducer.

Thus, there is no pressure lag error in straight, level, unaccelerated flight.

Definitions of Symbols

We define the following variables and constants of a general nature:

G force of gravity at mean sea level

32.17405 ft/sec^2
9.80665 m/sec^2

H scale height of the atmosphere 27672.24 ft.

h_p barometric altitude corrected for position and pressure lag error

$\overline{h}_p$ ADC-sensed barometric altitude

M_n true Mach number

P_o standard pressure

29.92 in. Hg.
101332.27 N/m^2
2116.2 lb/ft^2

P_s static pressure

P_t total pressure (sum of dynamic and static pressure)

q dynamic pressure (P_t - P_s)

R gas constant for dry air

1716.5 ft^2/sec^2-°R
287.04 m^2/sec^2-°K

T_o standard temperature

518.688 °R
288.16 °K

V_{vv} vertical velocity or rate of change in altitude

α standard temperature lapse rate of the atmosphere

0.00356616 °F/ft
0.0065 °C/m

α_T true angle of attack

γ ratio of specific heats
1.4 for air

τ time constant associated with the static pressure system

Derivation

The measured barometric altitude must be corrected for position error and pressure lag error. The functional forms of these quantities are determined by wind tunnel and flight testing. In the preceding section, it

was noted that position error is a function of speed, angle of attack, and altitude, while pressure lag error is a function of the time constant of the cavity. This can be expressed in the form,

$$h_p = \overline{h}_p + f_p\left(M_n, \alpha_T, \overline{h}_p\right) + f_\tau\left(\tau, V_{vv}\right) , \qquad (2\text{-}3)$$

where f_p denotes the functional form of position error and f_τ that of pressure lag error. The exact nature of both of these functions is determined by fitting curves through the experimental flight test data.

Now consider the pressure altitude only in terms of the parameters that form the position error, f_p, or

$$h_p \equiv h_p\left(M_n, \alpha_T\right) .$$

Then a Taylor series expansion of $h_p\left(M_n, \alpha_T\right)$ about $M_n = M_{n_o}$ and $\alpha_T = \alpha_{T_o}$ yields

$$h_p(M_n, \alpha_T) = h_p\left(M_{n_o}, \alpha_{T_o}\right) + \left(M_n - M_{n_o}\right)\left(\frac{\partial}{\partial M_n} h_p(M_n, \alpha_T)\right)_{M_{n_o}, \alpha_{T_o}}$$

$$+ \left(\alpha_T - \alpha_{T_o}\right)\left(\frac{\partial}{\partial \alpha_T} h_p(M_n, \alpha_T)\right)_{M_{n_o}, \alpha_{T_o}}$$

$$+ 0\left(\Delta M_n^{\,2}, \Delta\alpha_T^{\,2}\right) . \qquad (2\text{-}4)$$

We let

$$\bar{h}_p = h_p \left(M_{n_o}, \alpha_{T_o}\right) ,$$

and see that $\bar{h}_p$ can be treated as a constant.

Comparing Equation 2-4 with Equation 2-1, we see that the former will reduce to the latter if the position error is only a function of Mach number, and if a constant, k_1, is defined as

$$k_1 \equiv \left(\frac{\partial h_p}{\partial M_n}\right)_{M_{n_o}, \alpha_{T_o}} .$$

The resulting equation is

$$h_p = \bar{h}_p + k_1 \left(M_n - M_{n_o}\right) . \tag{2-5}$$

The constants, k_1 and M_{n_o}, are empirically determined from flight test data.

If a similar comparison is made between Equation 2-2 and Equation 2-4, no formal resemblance is noted. However, if the error in the measurement of true altitude due to position error is assumed to be a function of dynamic pressure (q) and Mach number (M_n) rather than speed, angle of attack, and altitude, then a Taylor series expansion of h_p (q, M_n) about $q = q_o$ and $M_n = M_{n_o}$ yields

$$h_p(q,M_n) = h_p(q_o, M_{n_o}) + (q - q_o) \left(\frac{\partial}{\partial q} h_p (q, M_n)\right)_{q_o, M_{n_o}}$$

$$+ (M_n - M_{n_o}) \left(\frac{\partial}{\partial M_n} h_p (q, M_n)\right)_{q_o, M_{n_o}}$$

$$+ 0 \left(\Delta q^2, \Delta M_n{}^2\right) . \qquad (2\text{-}6)$$

We let

$$\overline{h}_p = h_p (q_o, M_{n_o}) ,$$

and see that $\overline{h}_p$ can be treated as a constant.

Equation 2-6 must be related to flight-measurable quanitites. Consider Bernoulli's Equation,

$$\frac{q}{P_s} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_n{}^2\right]^{\left(\frac{\gamma}{\gamma - 1}\right)} -1 , \qquad (2\text{-}7)$$

and the defining equation for the standard atmosphere,

$$\frac{P_s}{P_o} = \left(\frac{T_o - \alpha h_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)} . \qquad (2\text{-}8)$$

Using $\bar{h}_p$ to approximate h_p in Equation 2-8, and substituting for P_s in Equation 2-7, yields

$$q \approx P_o \left(1 - \frac{\alpha \bar{h}_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)} \left\{\left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^2\right]^{\left(\frac{\gamma}{\gamma - 1}\right)} - 1\right\} , \qquad (2\text{-}9)$$

from which it is seen that we may consider $q \equiv q\,(M_n)$. Thus, by the chain rule,

$$\frac{\partial h_p}{\partial M_n} = \frac{\partial h_p}{\partial q} \frac{dq}{dM_n} ,$$

and so Equation 2-6 can be written as

$$h_p \approx \bar{h}_p + \left[(q - q_o) + (M_n - M_{n_o}) \frac{dq}{dM_n}\right] \left(\frac{\partial h_p}{\partial q}\right)_{q_o,\, M_{n_o}} . \qquad (2\text{-}10)$$

Taking derivatives in Equation 2-9, we have

$$\frac{dq}{dM_n} \approx P_o \gamma \left(1 - \frac{\alpha \bar{h}_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)} \left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^2\right]^{\left(\frac{1}{\gamma - 1}\right)} M_n . \qquad (2\text{-}11)$$

Since q_o is defined to be a reference dynamic pressure at the point where the correction is zero, it follows that $q_o = 0$ at $M_{n_o} = 0$. In view of this, and using Equations 2-9 and 2-11, we can write Equation 2-10 as

$$h_p \approx \bar{h}_p + \left\{P_o \left(1 - \frac{\alpha \bar{h}_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)} \left[\left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^2\right]^{\left(\frac{\gamma}{\gamma - 1}\right)} - 1\right]\right.$$

$$\left. + P_o \gamma \left(1 - \frac{\alpha \bar{h}_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)} \left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^2\right]^{\left(\frac{1}{\gamma - 1}\right)} M_n^2\right\} \left(\frac{\partial h_p}{\partial q}\right)_{o,o}$$

or

$$h_p \approx \bar{h}_p + P_o \left(1 - \frac{\alpha \bar{h}_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)} \left\{ \left[\left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^{\,2} \right]^{\left(\frac{\gamma}{\gamma - 1}\right)} - 1 \right] \right.$$

$$\left. + \gamma \left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^{\,2} \right]^{\left(\frac{1}{\gamma - 1}\right)} M_n^{\,2} \right\} \left(\frac{\partial h_p}{\partial q}\right)_{o,o} . \qquad (2\text{-}12)$$

Since $\left| \frac{\alpha \bar{h}_p}{T_o} \right| < 1$ and $\left| \left(\frac{\gamma - 1}{2}\right) M_n^{\,2} \right| < 1$ for all reasonable values, the binomial theorem may be used to generate the sums

$$\left(1 - \frac{\alpha \bar{h}_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)} = \sum_j \binom{\frac{G}{R\alpha}}{j} \left(- \frac{\alpha \bar{h}_p}{T_o}\right)^j , \qquad (2\text{-}13)$$

$$\left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^{\,2} \right]^{\left(\frac{1}{\gamma - 1}\right)} = \sum_j \binom{\frac{1}{\gamma - 1}}{j} \left[\left(\frac{\gamma - 1}{2}\right) M_n^{\,2} \right]^j , \qquad (2\text{-}14)$$

and

$$\left[1 + \left(\frac{\gamma - 1}{2}\right) M_n^{\,2} \right]^{\left(\frac{\gamma}{\gamma - 1}\right)} = \sum_j \binom{\frac{\gamma}{\gamma - 1}}{j} \left[\left(\frac{\gamma - 1}{2}\right) M_n^{\,2} \right]^j . \qquad (2\text{-}15)$$

Actually applying Equations 2-13, 2-14, and 2-15 to Equation 2-12, and neglecting resultant terms containing α or M_n^j for $j \geq 4$, yields

$$h_p \approx \bar{h}_p \left[1 - M_n^{\,2} \left(K_1 - K_2 \, \bar{h}_p \right) \right] \qquad (2\text{-}16)$$

where

$$K_1 = -\frac{3}{2} P_o \gamma \left[\frac{\partial}{\partial q} \left(\frac{h_p}{\bar{h}_p} \right) \right]_{o,o}$$

and

$$K_2 = \frac{G}{RT_o} K_1 \quad .$$

The form of Equation 2-16 now matches that of Equation 2-2. The constants, K_1 and K_2, depend on the empirically determined value of the derivative. This derivative - or more likely, the values of K_1 and K_2 - are determined by flight and wind tunnel testing. The constant K_1 is unitless, while K_2 has units of length. Rewriting Equation 2-16 in a slightly different form gives

$$h_p \approx \bar{h}_p \left[1 - M_n^2 \left(K_1 - K_2' \frac{\bar{h}_p}{H} \right) \right] . \qquad (2\text{-}17)$$

The constant, H, introduced in Equation 2-17 is the scale height of the atmosphere,

$$H \equiv \frac{T_o R}{G} \quad .$$

To five decimal places, the constants are

$$K_1 = -\frac{3}{2} \gamma P_o \left[\frac{\partial}{\partial q} \left(\frac{h_p}{\bar{h}_p} \right) \right]_{o,o} = .02032$$

and

$$K_2' = -\frac{3}{2}\gamma P_o \left[\frac{\partial}{\partial q}\left(\frac{h_p}{\bar{h}_p}\right)\right]_{o,o} = .02080 .$$

The difference in the values, K_1 and K_2', is due to the fact that the original constants were empirically determined. A simplified form of Equation 2-17 is possible if a single value is used for both constants. This result is

$$h_p \approx \bar{h}_p \left[1 - K_1' \; M_n^2 \left(1 - \frac{\bar{h}_p}{H}\right)\right], \tag{2-18}$$

with

$$K_1' = .02032 .$$

The next step is to include the effects of pressure lag error. Assumption of a linear dependence of altitude correction on vertical velocity, V_{vv}, gives

$$f_\tau = -K_3 \; V_{vv} + c . \tag{2-19}$$

Since there is no correction in level flight, the constant, c, is zero. The constant, K_3, has dimensions of time and is the time constant of the cavity. The non-AIMS system is not corrected for the cavity time constant, while the AIMS-system correction becomes

$$h_p = \bar{h}_p \left[1 - M_n^2 \left(K_1 - K_2' \; \frac{\bar{h}_p}{H}\right)\right] - K_3 \; V_{vv} , \tag{2-20}$$

where K_1 and $K_2^{'}$ are defined as above and

$$K_3 = .4375 \text{ sec.}$$

The equations for the position and pressure lag errors are dependent on the Pitot-static system used, the physical dimensions of that system, and the standard atmosphere assumed.

The only remaining correction is the offset applied to the value of altitude reported by the pressure transducer. The OFP uses an offset of -1024 feet (a convenient power of two), while the assumed offsets for the non-AIMS-probe system and the AIMS-probe system are -1020 feet and -1056 feet, respectively. The final form of the equation for non-AIMS-probe should then be

$$h_p = \bar{h}_p + K_1 \left(M_n - M_{n_o}\right) - K_2 . \qquad (2\text{-}21)$$

The final form of the equation for the AIMS-probe is

$$h_p = \bar{h}_p \left[1 - M_n^{\,2}\left(K_1 - K_2^{'}\ \frac{\bar{h}_p}{H}\right)\right] - K_3\ V_{vv} - K_4 . \qquad (2\text{-}22)$$

The values and physical meanings of the constants appearing in Equation 2-22 are given in Table 2.1.

Table 2.1 LOCAL CONSTANTS

Symbol	Physical Meaning	Dependencies	Value/Units
k_1	instantaneous rate of change of ratio of sensed barometric altitudes and Mach number	altimeter	560 feet
k_2	Non-AIMS offset	OFP	1020 feet
M_{n_o}	Mach number offset	empirical testing	.2 unitless
K_1	$= -\frac{3}{2}\gamma P_o \left[\frac{\partial}{\partial q}\left(\frac{h_p}{\bar{h}_p}\right)\right]_{o,o}$	empirical testing	.0232 unitless
K_2	$= -\frac{3}{2}\left(\frac{G}{RT_o}\right)\gamma P_o \left[\frac{\partial}{\partial q}\left(\frac{h_p}{\bar{h}_p}\right)\right]_{o,o}$	empirical testing	.02080 unitless
K_3	time constant	empirical testing	.4375 sec
K_4	AIMS offset	OFP	1056 feet

2.3 Correction to Altitude for Non-Standard Sea Level Pressure

Background

Page ADC-13 of Reference 1 on the Air Data Computer gives the equation,

$$Pl := cfeet\_dist \{alt + [cingh(slp - ingh(29.92))] * [924.87 - .00635856*alt]\} \quad . \qquad (2\text{-}23)$$

This equation corrects for a non-standard sea level pressure (i.e. a pressure other than 29.92 inches Hg at sea level, and set into the computer by the pilot).

There are several important terms that are relevant to the measurement of altitude.

Tape line altitude - The actual altitude that an hypothetical ruler or tape would measure from mean sea level to the aircraft. This altitude is that which would be given by a perfect altimeter.

Geopotential altitude - The measure of the potential energy of a unit mass at this point relative to mean sea level. The geopotential altitude is equal to the tape line altitude only if the force of gravity is not a function of altitude. Geopotential altitude must be considered in all pressure or density calculations.

Pressure Altitude - The altitude given by an aneroid altimeter. It is a measure of the difference in pressure between a standard datum (usually mean sea level) and the measured static pressure converted to distance by assuming a standard atmosphere.

The problem encountered here is that the pressure altitude furnished to the ADC is determined using the standard atmosphere, or 29.92 inches Hg. This pressure may or may not correspond to that encountered at mean sea level. The required altitude is that determined using the actual pressure at mean sea level.

Definitions of Symbols

We define the following variables and constants of a general nature:

G acceleration due to gravity at mean sea level

32.17405 ft/sec^2
9.80665 m/sec^2

$g(z)$ acceleration due to gravity, a function of altitude

h_p geopotential altitude (see Figure 2.1)

H scale height of the atmosphere 27672.24 ft.

$\overline{H}_{ph}$ pressure altitude corrected for actual pressure at sea level (see Figure 2.1)

P_o standard pressure (see Figure 2.1)

29.92 In Hg
101332.27 N/m^2
2116.2 lb/ft^2

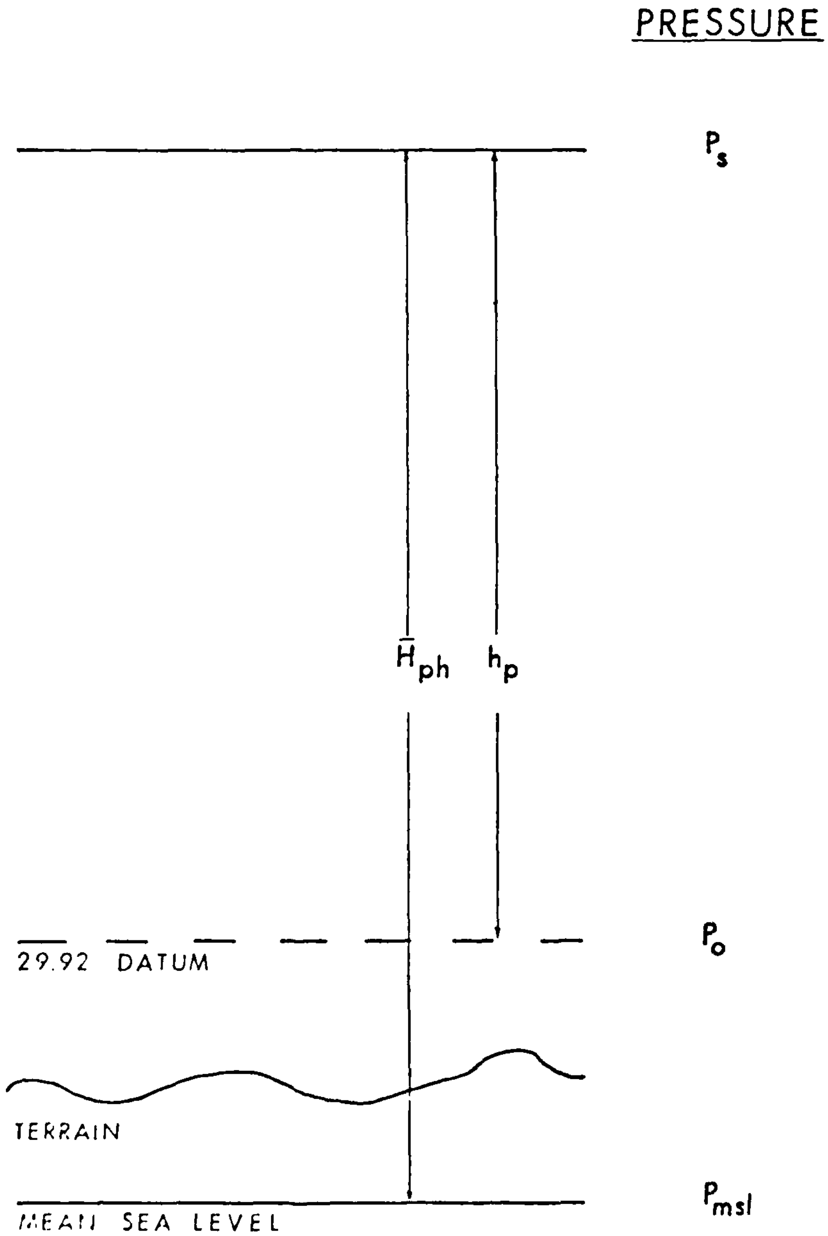

Figure 2.1 Altitude Measurement Definitions

P_s measured value of the static pressure

R gas constant for dry air

1716.5 ft^2/sec^2-°R
287.04 m^2/sec^2-°K

T_o standard temperature

518.688 °R
288.16 °K

z tape line altitude, i.e., the actual distance from mean sea level to the aircraft

α standard temperature lapse rate of the atmosphere

0.00356616 °F/ft
0.0065 °C/m

ΔP difference between standard pressure and actual pressure at sea level

Derivation

To arrive at Equation 2-23, consider a Taylor series expansion about the altitude (h_p) computed using the standard pressure datum (P_o), which yields

$$\overline{H}_{ph} = h_p + \Delta P \frac{\partial h_p}{\partial P_o} + O(\Delta P^2). \tag{2-24}$$

If terms of the order of ΔP^2 and higher are neglected, then the remaining problem is to determine $\partial h_p/\partial P_o$. The standard atmosphere is given by the equation,

$$h_p = \frac{T_o}{\alpha}\left[1 - \left(\frac{P_s}{P_o}\right)^{(\alpha R/G)}\right]. \qquad (2\text{-}25)$$

Since the static pressure, P_s, is not known to the module, it is necessary to calculate it based on the pressure altitude and other known sea level quantities. The derivation begins with the definition of the temperature lapse rate, α, or

$$\frac{dT}{dh_p} = -\alpha. \qquad (2\text{-}26)$$

Both sides of Equation 2-26 are multiplied by G/RT to give

$$\frac{G}{RT}\,\frac{dT}{dh_p} = -\,\frac{G\alpha}{RT}. \qquad (2\text{-}27)$$

Geopotential altitude is defined as a measure of the gravitational potential energy of a unit mass at a point relative to mean sea level. It can be defined in differential form by the equation,

$$G\,dh_p = g(z)dz. \qquad (2\text{-}28)$$

Equations 2-27 and 2-28 along with the chain rule are used to give

$$\frac{G}{RT}\,\frac{dT}{dz}\,\frac{dz}{dh_p} = -\,\frac{G\alpha}{RT},$$

or

$$\frac{G}{RT}\frac{dT}{dz} = -\frac{g(z)\alpha}{RT} \; . \qquad (2\text{-}29)$$

Next, use is made of the following assumptions: (1) the air is dry, (2) the atmosphere is a perfect diatomic gas, and (3) the atmosphere is in equilibrium.

The atmosphere then obeys the perfect gas law,

$$P = \rho RT, \qquad (2\text{-}30)$$

where ρ is the density of the atmosphere.

For the case of hydrostatic equilibrium, it can be assumed that

$$\frac{dP}{dz} = -\rho g(z). \qquad (2\text{-}31)$$

In view of Equation 2-31, Equation 2-29 can be written (again using the chain rule) as

$$\frac{G}{RT}\frac{dT}{dP}\frac{dP}{dz} = -\frac{g(z)\alpha}{RT} \; .$$

Use of these results and Equation 2-31 yields

$$\frac{G}{RT}\frac{dT}{dP}\left(-\rho g(z)\right) = -\frac{g(z)\alpha}{RT} \; ,$$

or

$$\frac{G}{R\alpha T}\frac{dT}{dP} = \frac{1}{P} \; .$$

Thus we have

$$\frac{G}{R\alpha} \frac{dT}{T} = \frac{dP}{P} . \tag{2-32}$$

Now, integration of the defining equation for the temperature lapse rate (Equation 2-26) gives

$$\int_{T_o}^{T} dT' = -\alpha \int_{o}^{h_p} dh_p'$$

or

$$T = T_o - \alpha h_p . \tag{2-33}$$

Integration of Equation 2-32 from mean sea level to the altitude in question yields

$$\frac{G}{R\alpha} \int_{T_o}^{T} \frac{dT'}{T'} = \int_{P_o}^{P_s} \frac{dP'}{P'} ,$$

or

$$\frac{G}{R\alpha} \ell n \left({}^{T}/_{T_o} \right) = \ell n \left({}^{P_s}/_{P_o} \right) . \tag{2-34}$$

Equation 2-33 is substituted into Equation 2-34 with the result,

$$\ell n \left(\frac{P_s}{P_o} \right) = \frac{G}{R\alpha} \ell n \left(\frac{T_o - \alpha h_p}{T_o} \right) ,$$

or

$$\frac{P_s}{P_o} = \left[\frac{T_o - \alpha h_p}{T_o}\right]^{\left(\frac{G}{R\alpha}\right)} . \tag{2-35}$$

Differentiation of Equation 2-25 with respect to P_o gives

$$\frac{\partial h_p}{\partial P_o} = - \frac{T_o}{\alpha} \left(P_s\right) \left(\frac{\alpha R}{G}\right) \frac{\partial}{\partial P_o} \left[\left(\frac{1}{P_o}\right)^{\left(\frac{\alpha R}{G}\right)}\right] ,$$

or

$$\frac{\partial h_p}{\partial P_o} = \left[\frac{T_o R}{P_o G} \left(\frac{P_s}{P_o}\right)^{\left(\frac{\alpha R}{G}\right)}\right] . \tag{2-36}$$

Equation 2-35 is now substituted into Equation 2-36 to obtain

$$\frac{\partial h_p}{\partial P_o} = \frac{T_o R}{P_o G} \left[\left(\frac{T_o - \alpha h_p}{T_o}\right)^{\left(\frac{G}{R\alpha}\right)}\right]^{\left(\frac{\alpha R}{G}\right)} ,$$

or

$$\frac{\partial h_p}{\partial P_o} = \frac{T_o R}{P_o G} \left(1 - \frac{\alpha h_p}{T_o}\right) .$$

This equation can be written in the form,

$$\frac{\partial h_p}{\partial P_o} = \frac{T_o R}{P_o G} - \frac{\alpha R}{P_o G} h_p . \tag{2-37}$$

Substitution of this result into Equation 2-24 and use of $\Delta P = (P_{msl} - P_o)$ gives

$$\overline{H}_{ph} = h_p + (P_{msl} - P_o)\left(\frac{T_o R}{P_o G} - \frac{\alpha R}{P_o G}\; h_p\right). \qquad (2\text{-}38)$$

If the constants are now evaluated, Equation 2-38 becomes

$$\overline{H}_{ph} = h_p + (P_{msl} - 29.92 \text{ in Hg})\,[924.87 \text{ ft/in Hg} - .006358831/\text{in Hg}\,(h_p)] \quad . \qquad (2\text{-}39)$$

Equation 2-39 now agrees with Equation 2-23 except for a small difference in the seventh decimal place in the third constant.

The constants appearing in Equation 2-39 are not grouped in a manner that makes their aggregate value independent of the unit system chosen. In Equation 2-38, P_o can be factored out to give

$$\overline{H}_{ph} = h_p + \left(\frac{P_{msl}}{P_o} - 1\right)\left(H - A\,h_p\right), \qquad (2\text{-}40)$$

where

$$H \equiv \frac{T_o R}{G} \quad \text{and}$$

$$A \equiv \frac{\alpha R}{G} \; .$$

The constant, H, (which has dimensions of length) is called the Scale Height and is defined to be the height at which the pressure of an isothermal atmosphere at temperature, T_o, decays to e^{-1} of its surface value. The

non-dimensional constant, A, represents the temperature lapse rate made non-dimensional by scale height, H, and standard surface temperature, T_0. These results are

$$H = \left(518.688\ ^\circ R\right)\left(\frac{1716.5\ ft^2}{sec^2\ ^\circ R}\right)\left(\frac{sec^2}{32.17405\ ft}\right)$$

or

$$H = 27672.24\ ft$$

and

$$A = \left(\frac{.00356616\ ^\circ F}{ft}\right)\left(\frac{1716.5\ ft^2}{sec^2\ ^\circ R}\right)\left(\frac{1\ ^\circ R}{1\ ^\circ F}\right)\left(\frac{sec^2}{32.17405\ ft}\right)$$

or

$$A = 0.1902562.$$

The three constants appearing in Equation 2-40 are dependent only on the units used for the standard atmosphere. The equation itself is not aircraft dependent: its form will change only if the standard atmosphere is changed or the underlying assumptions used in the derivation are changed. The values and physical meaning of the constants appearing in Equation 2-40 are given in Table 2.2.

Table 2.2 LOCAL CONSTANTS

Symbol	Physical Meaning	Dependencies	Value/Units
A	temperature lapse rate made non-dimensional by scale height and standard surface temperature	standard atmosphere	0.1902562 unitless
H	scale height of the atmosphere	standard atmosphere	27672.24 ft.

2.4 References

1. "NRL A7 Software Project Working Paper 6150a, Version 8," 22 June 1981.

2. "NRL A7 Software Project Working Paper 6603a, Version 2," 15 July 1981.

SECTION 3
ANGLE OF ATTACK (AOA)

3.1 General

The Angle of Attack (AOA) calculations are performed by converting the input value of the indicated AOA to its true value and smoothing these converted values.

3.2 AOA Scaling and Smoothing

Background

In Reference 1, the OFP listing equation for final smoothed AOA, $\tilde{\alpha}$, is given as (read "=" as meaning "set equal to")

$$\tilde{\alpha} = \approx\!\!\!\alpha = \left(\approx\!\!\!\alpha - 0.07842\ \approx\!\!\!\alpha\right) + \left(\alpha'_p + \alpha'_{IN}\right), \qquad (3\text{-}1)$$

where $\approx\!\!\!\alpha$ is a smoothed AOA. It is given that

$$\alpha'_p = 0.03921\left[(1.598155\mathrm{x}10^{-5})\alpha_p - (0.0241111)\right]$$

and

$$\alpha'_{IN} = 0.03921\left[(1.598155\mathrm{x}10^{-5})\alpha_{IN} - (0.0241111)\right],$$

where α_p and α_{IN} are the two different indicated AOA sample values.

As can be seen, Equation 3-1 is rife with numerical constants and confusing notation.

The AOA system consists of a movable vane located on the left side of the fuselage near the nose of the aircraft. The vane senses the angle between the free stream movement of the air (the velocity vector) and an arbitrary fuselage reference line. The AOA vane is immersed in the turbulent flow about the aircraft and, having a much lower effective mass, responds more quickly to inputs than the aircraft. The quick response of the vane produces noise on top of the actual AOA signal produced by the transducer. The signal must be filtered to remove the noise, and it must be scaled and offset to indicate the appropriate angle.

Definition of Variables

We define the following variables and general constants:

T	computation time period
α	represents α_{IN} or α_p
$\tilde{\alpha}$	final smoothed AOA
$\approx$ over α	smoothed AOA
α_I	two input AOA values (α_{IN} and α_p) averaged over fixed sampling period
α_{IN}	first indicated AOA, scaled and formatted, within fixed sampling period
α_p	second and final indicated AOA, scaled and formatted, within fixed sampling period
α_T	true AOA
α'_{IN}	true AOA times a constant (see α_{IN})

α_p' true AOA times a constant (see α_p)

$\hat{\alpha}_{IN}$ indicated AOA, formatted (see α_{IN})

$\hat{\alpha}_p$ indicated AOA, formatted (see α_p)

Derivation

The transfer function of the AOA system is given to be

$$\frac{\tilde{\tilde{\alpha}}}{\alpha_I} = \frac{1}{\tau s + 1} \quad , \tag{3-2}$$

where τ is a time constant that has been empirically determined. Equation 3-2 can be written as

$$\tilde{\tilde{\alpha}} s = \tau^{-1} \left(\alpha_I - \tilde{\tilde{\alpha}}\right) \quad . \tag{3-3}$$

Since the inputs to the AOA system, α_{IN} and α_p, are sampled over a period, T, application of a Z-transform is suitable (see Reference 2, Appendix A; also see References 3 and 4).

Let

$$z = e^{sT} \quad , \tag{3-4}$$

where z is actually the Z-transform operator. Clearly we have

$$z^{-1} = e^{-sT} \quad . \tag{3-5}$$

Since an exponential can be written in the form,

$$e^{p} = \sum_{j \geq 0} \frac{(p)^{j}}{j!} \quad , \tag{3-6}$$

we take

$$z^{-1} = 1 - sT \quad , \tag{3-7}$$

or

$$s = \frac{(1 - z^{-1})}{T} \quad . \tag{3-8}$$

Thus, Equation 3-3 can be written

$$\tilde{\tilde{\alpha}} \left(\frac{1 - z^{-1}}{T} \right) = \tau^{-1} \left(\alpha_I - \tilde{\tilde{\alpha}} \right) \quad . \tag{3-9}$$

Actually applying the z-operator in Equation 3-9 yields

$$\frac{\tilde{\tilde{\alpha}}_n - \tilde{\tilde{\alpha}}_{n-1}}{T} = \tau^{-1} \left(\alpha_I - \tilde{\tilde{\alpha}}_n \right) \quad , \tag{3-10}$$

or

$$\left(1 + \frac{T}{\tau} \right) \tilde{\tilde{\alpha}}_n = \tilde{\tilde{\alpha}}_{n-1} + \left(\frac{T}{\tau} \right) \alpha_I \quad . \tag{3-11}$$

In view of how α_I is defined, we multiply both sides by $(1 + T/\tau)^{-1}$. Equation 3-11 can then be written

$$\tilde{\tilde{\alpha}}_n = \left(1 + \frac{T}{\tau} \right)^{-1} \tilde{\tilde{\alpha}}_{n-1} + \left(1 + \frac{\tau}{T} \right)^{-1} \left(\frac{\hat{\alpha}_{IN} + \hat{\alpha}_p}{2} \right) \tag{3-12}$$

or

$$\tilde{\tilde{\alpha}}_n = \left(1 + \frac{T}{\tau}\right)^{-1} \tilde{\tilde{\alpha}}_{n-1} + \frac{1}{2}\left(1 + \frac{\tau}{T}\right)^{-1} \left(\hat{\alpha}_{IN} + \hat{\alpha}_p\right) . \qquad (3\text{-}13)$$

To get Equation 3-13 to include the arithmetic for indicated AOA, we note that the OFP is supplied AOA from the AOA vane. The geometry of the AOA vane is illustrated in Figure 3-1.

From Figure 3-1, we see that

$$\alpha(\text{electrical AOA}) = \alpha_I + \theta . \qquad (3\text{-}14)$$

The position error equation has been calculated by systems engineers to be

$$\alpha_T = k_1 \alpha_I + k_2 . \qquad (3\text{-}15)$$

Substituting the value of indicated AOA from Equation 3-14, we get

$$\alpha_T = k_1 (\alpha - \theta) + k_2 , \qquad (3\text{-}16)$$

or

$$\alpha_T = k_1\alpha - k_1\theta + k_2 . \qquad (3\text{-}17)$$

We need Equation 3-17 to give AOA to the Arbitrary Datum Line (ADL) and not waterline - 100. Therefore, we need

$$\alpha_T = k_1\alpha - k_2' , \qquad (3\text{-}18)$$

where

$$k_2' = k_1\theta + k_2 - \Psi .$$

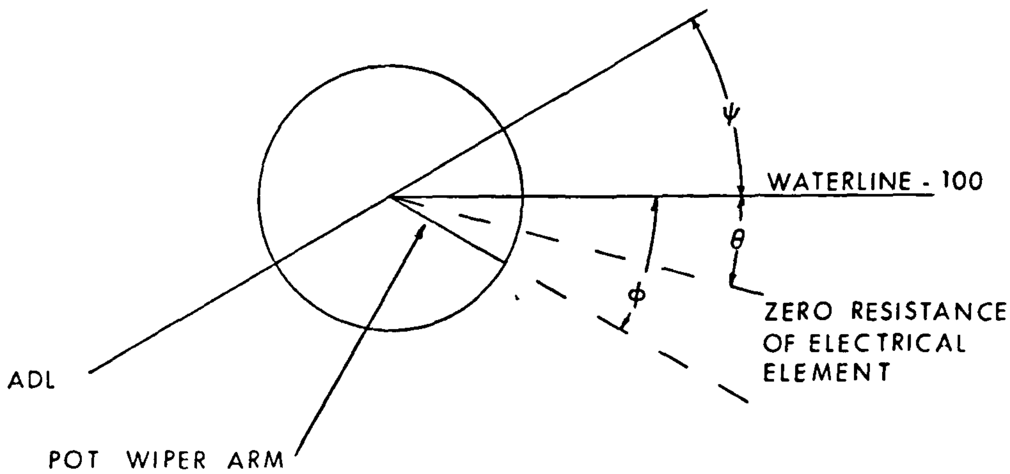

Figure 3-1. Diagram of Angle of Attack Vane

The Signal Data Converter (SDC) scales and formats the DC voltage input by an angular constant, λ. Further, the AOA is scaled by an angular constant, ω, to effect a conversion to OFP angular coordinates. Thus, Equation 3-18 becomes

$$\alpha_T = \frac{k_1\lambda}{\omega}\alpha - \frac{k_2'}{\omega} , \tag{3-19}$$

from which we have

$$\hat{\alpha}_{IN} = K_1\, \alpha_{IN} - K_2 \tag{3-20}$$

and

$$\hat{\alpha}_p = K_1\, \alpha_p - K_2 , \tag{3-21}$$

where

$$K_1 = \frac{k_1\lambda}{\omega}$$

and

$$K_2 = \frac{k_2'}{\omega} .$$

With the results,

$$\alpha_{IN}' = \frac{1}{2}\left(1 + \frac{\tau}{T}\right)^{-1} \hat{\alpha}_{IN} \tag{3-22}$$

and

$$\alpha_p' = \frac{1}{2}\left(1 + \frac{\tau}{T}\right)^{-1} \hat{\alpha}_p , \tag{3-23}$$

Equation 3-13 may be written

$$\tilde{\alpha} = \left(\approx{\alpha} - \kappa\approx{\alpha}\right) + \left(\alpha'_{IN} + \alpha'_{p}\right) \tag{3-24}$$

where

$$\kappa = 1 - \left(1 + \frac{T}{\tau}\right)^{-1} ,$$

$$\alpha'_{IN} = \frac{1}{2}\left(1 + \frac{\tau}{T}\right)^{-1} \left(K_1\, \alpha_{IN} - K_2\right) ,$$

and

$$\alpha'_{p} = \frac{1}{2}\left(1 + \frac{\tau}{T}\right)^{-1} \left(K_1\, \alpha_{p} - K_2\right) .$$

The values of the constants are given in Table 3.1.

Table 3.1 LOCAL CONSTANTS

Symbol	Physical Meaning	Dependencies	Value/Units
k_1	none	AOA sensor and test data	0.785525, 0.76 unitless
k_2	none	AOA sensor and test data	0.4 degrees
τ	time constant	function of weapons rate	0.47007 sec
T	weapons rate	function of filter constant	0.04 sec
θ	angle between WL-100 and zero resistance of electrical moment	AOA vane	8 degrees
$\emptyset$	angle between WL-100 and pot wiper arm	AOA vane	9 degrees
ψ	angle between ADL and WL-100	AOA vane	3 degrees
λ	scales DC voltage input	SDC	30/4096 degrees
ω	conversion constant to change units to circles	units to be converted are in degrees	360 degrees

3.3 References

1. Crews, L. L., and Hall, C. W., "A-7D/E Aircraft Navigation Equations," Naval Weapons Center, Technical Note 404-176, March 1975.

2. George, R.G., "A-7E Aircraft Weapon Delivery Equations," Naval Weapons Center, NWC Technical Memorandum 2926, September 1976.

3. Dorf, R. C., "Modern Control Systems," 3rd ed., Addison-Wesley Publishing Co., 1976.

4. Churchill, R. V., "Operational Mathematics," 3rd ed., McGraw-Hill, Inc., 1944.

SECTION 4
DOPPLER RADAR SET

4.1 General

The Doppler Radar System is a self-contained dead-reckoning system that obtains desired navigational information involving aircraft velocity by means of a Doppler Radar and direction by means of a directional sensor.

4.2 Corrections to Doppler Ground Speed

Background

Page DRS-5 of Reference 2 on the Doppler Radar Set gives the equation,

$$\text{Pl:= ground - (.00025*ground*alt)/1000.} \qquad (4\text{-}1)$$

This equation corrects the Doppler ground speed to account for earth curvature and other empirically determined inaccuracies which are functions of altitude.

Figure 4.1 illustrates flight over curved earth. The aircraft is located at O, with lift, drag, and weight vectors as indicated. The flight path makes an angle, γ, with the horizontal, and the aircraft moves

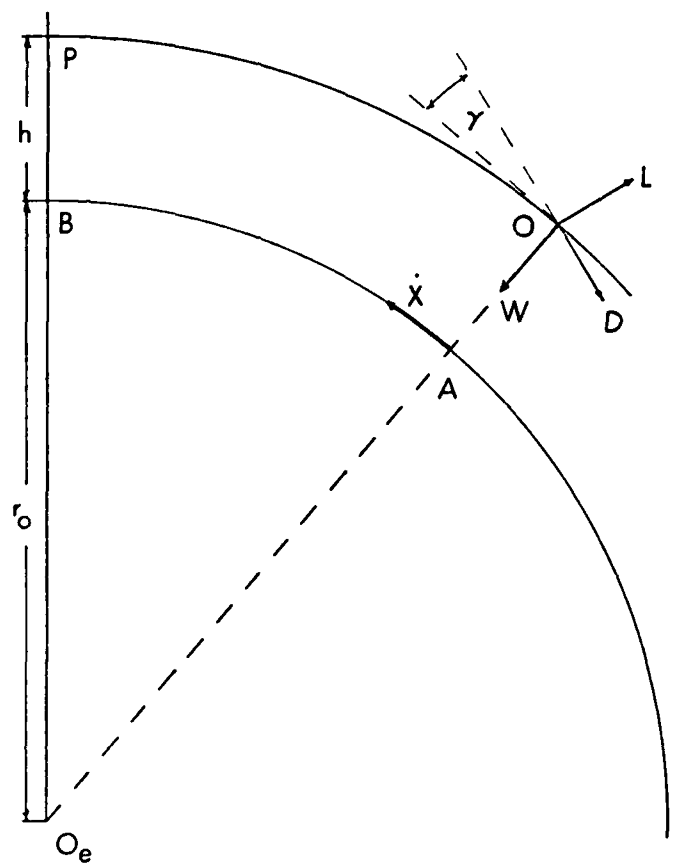

Figure 4.1 Earth Centered Coordinate System

along the path, OP. The line joining 0 to 0_e (the center of the earth) sweeps along the earth's surface with a velocity, X (the ground speed). The measured value of velocity in the aircraft frame of reference is greater than that described at the earth's surface. Thus, the measured value of the ground speed must be corrected for aircraft altitude.

The Doppler radar system contains other errors that are functions of aircraft altitude or radar slant range. The aggregate value of these errors has been determined by flight testing the aircraft at various altitudes and comparing the measured velocities with photographically derived data. The curve that was fitted through these data points is described by Equation 4-1.

Definition of Variables

We define the following variables and general constants:

A point of intersection of line 0_e0 with the earth's surface

D drag vector

h height of 0 above A

L lift vector

0_e origin of the earth centered coordinates

0 origin of the aircraft frame of reference

r_0 radius of the earth

V magnitude of the aircraft velocity vector in the earth centered coordinate system

W aircraft weight vector

$\dot{X}$ ground speed

γ angle measured from the horizontal to the velocity vector

Derivation

The equation of motion for flight in a great circle plane is

$$\dot{X} = V \frac{r_o}{r_o + h} \cos \gamma, \tag{4-2}$$

where V is the velocity vector of the aircraft in the earth centered coordinate system. If γ = o, which implies flight along a path parallel to the earth's surface, Equation 4-2 becomes

$$\dot{X} = V \left(1 + {}^{h}/_{r_o}\right)^{-1} . \tag{4-3}$$

The right hand side of Equation 4-3 can be expanded with the binomial series to yield

$$\dot{X} = V \sum_{j \geq 0} \left(- \frac{h}{r_o}\right)^{j} .$$

Since the quantity, h/r_o, is very small, terms of second and higher powers may safely be neglected, thus resulting in

$$\dot{X} = V - \left({}^{h}/_{r_o}\right) V . \tag{4-4}$$

If the value for r_o is substituted into Equation 4-4 in the same way as in Equation 4-1, the result is

$$\dot{X} = V - KhV/1000 . \tag{4-5}$$

The value given in Table 4.1 for K is about 1/5 of that used in Equation 4-1. The difference can be accounted for by the fact that the value of K represents not only that correction due to flight over a curved earth but also a number of empirically determined correction factors. To proceed further with an analytic derivation of the given value of K would involve a detailed analysis of the specific Doppler radar on the A-7 aircraft.

Finally, we need to pose either Equation 4-1 or Equation 4-5 in a form which contains constants of the correct variable type. The best form should be

$$\dot{X} = V\left(1 - {}^{h}/_{r_o}\right), \qquad (4\text{-}6)$$

with r_o = 20,930,000 ft.

Table 4.1 LOCAL CONSTANTS

Symbol	Physical Meaning	Dependencies	Value/Units
K	correction due to flight over curved earth plus altitude correction factors	set of flight test data used	4.8473×10^{-5} ft^{-1}

4.3 References

1. "NRL A7 Software Project Working Paper 6150a, Version 8," 22 June 1981.

2. "NRL A7 Software Project Working Paper 6603a, Version 2," 15 July 1981.

BIBLIOGRAPHY

1. Bachinski, F. A., "A-7E Operational Flight Program (OFP) Validation Plan and Procedures for Simulation Facilities OFP NWC-2: Part 1. Body of Text and Appendix A," Naval Weapons Center, NWC Technical Memorandum 3204, Part 1, March 1977.

2. Bachinski, F. A., "A-7E Operational Flight Program (OFP) Validation Plan and Procedures for Simulation Facilities OFP NWC-2: Part 2. Procedures," Naval Weapons Center, NWC Technical Memorandum 3204, Part 2, March 1977.

3. Basili, V. R., and Weiss, D. M., "Evaluation of a Software Requirements Document by Analysis of Change Data," Naval Research Laboratory, 1981.

4. Britton, K. H., and Parnas, D. L., "A-7E Software Module Guide," Naval Research Laboratory, NRL Memorandum Report 4702, 8 December 1981.

5. Churchill, R. V., "Operational Mathematics," 3rd ed., McGraw-Hill, Inc., 1944.

6. Crews, L. L., and Hall, C. W., "A-7D/E Aircraft Navigation Equations," Naval Weapons Center, Technical Note 404-176, March 1975.

7. Dorf, R. C., "Modern Control Systems," 3rd ed., Addison-Wesley Publishing Co., 1976.

8. Duke, A. A., Brown, T. H., Burke, K. W., and Seeley, R. B., "A Ballistic Trajectory Algorithm for Digital Airborne Fire Control," Naval Weapons Center, NWC Technical Publication 5416, September 1972.

9. Eshbach, O. W., ed., and Sounders, M., ed., "Handbook of Engineering Fundamentals," 3rd ed., John Wiley & Sons, Inc., 1936.

10. George, R. G., "A-7E Aircraft Weapon Delivery Equations," Naval Weapons Center, NWC Technical Memorandum 2926, September 1976.

11. Heninger, K. L., Kallander, J. W., Parnas, D. L., and Shore, J. E., "Software Requirements for the A-7E Aircraft," Naval Research Laboratory, NRL Memorandum Report 3876, 27 November 1978.

12. Kayton, M., ed., and Fried, W. R., ed., "Avionics Navigation Systems," John Wiley & Sons, Inc., 1969.

13. Korn, G. A., and Korn, T. M., "Mathematical Handbook for Scientists and Engineers," 2nd ed., rev., McGraw-Hill, Inc., 1961.

14. Mechtly, E. A., "The International System of Units: Physical Constants and Conversion Factors," rev., Office of Technology Utilization, National Aeronautics and Space Administration, NASA SP-7012, 1969.

15. Minzner, R. A., Champion, K. S. W., and Pond, H. L., "The ARDC Model Atmosphere.1959," Air Force Surveys in Geophysics (No. 115), AFCRC-TR-59-267, August 1959.

16. Parker, R. A. Heninger, K. L. Parnas, D. L., and Shore, J. E., "Abstract Interface Specifications for the A-7E Device Interface Module," Naval Research Laboratory, NRL Memorandum Report 4385, 20 November 1980.

17. Parnas, D. L., and Shore, J. E., "Language Facilities for Supporting the Use of Data Abstractions in the Development of Software Systems," Information Systems Staff, Naval Research Laboratory, 20 February 1978.

18. Von Mises, R., "Theory of Flight," Dover Publications, Inc., New York, 1959.

19. "A-7E Operation Flight Program Flow Charts Description," Vought Systems Division, LTV Aerospace Corporation.

20. "A-7E Operational Flight Program Math Flow Description," Vought Systems Division, LTV Aerospace Corporation.

21. "NATOPS Flight Manual: Navy Models A-7C, A-7E Aircraft," Direction of the Commanders, Naval Air Systems Command, NAVAIR 01-45AAE-1, 1 May 1977.

22. "NRL A7 Software Project Working Paper 6603a, Version 2," 15 July 1981.

23. "NRL A7 Software Project Working Paper 6759a, Version 4," 21 September 1981.

24. "NRL A7 Software Project Working Paper 6150a, Version 8," 22 June 1981.

APPENDIX A

LIST OF SYMBOLS

G	acceleration due to gravity at mean sea level 32.17405 ft/sec^2 9.80665 m/sec^2
$g(z)$	acceleration due to gravity as a function of altitude
h_p	geopotential altitude
$\bar{h}_p$	ADC-sensed barometric altitude
H	scale height of the atmosphere 27672.24 ft.
$\bar{H}_{ph}$	pressure altitude corrected for actual pressure at sea level
M_n	true Mach number
P_o	standard pressure 29.92 in. Hg. 101332.27 N/m^2 2116.2 lb/ft^2
P_s	static pressure
P_t	total pressure
q	dynamic pressure, $P_t - P_s$

R	the gas constant for dry air 1716.5 ft^2/sec^2-°R 287.04 m^2/sec^2-°K
T	computation time period
T_o	standard temperature 518.688 °R 288.16 °K
V_{vv}	vertical velocity or rate of change in altitude
z	tape line altitude, i.e., the actual distance from mean sea level to the point
α	standard temperature lapse rate of the atmosphere 0.00356616 °F/ft 0.0065 °C/m
$\tilde{\alpha}$	final smoothed Angle of Attack
$\approx$ α	smoothed Angle of Attack
α_I	two input Angle of Attack values (α_{IN} and α_p) averaged over fixed sampling period

LIST OF SYMBOLS (Continued)

α_{IN}	indicated Angle of Attack, scaled and formatted
α_p	indicated Angle of Attack, scaled and formatted
α_T	true Angle of Attack
α'_{IN}	true Angle of Attack times a constant
α'_p	true Angle of Attack times a constant
$\hat{\alpha}_{IN}$	indicated Angle of Attack, formatted
$\hat{\alpha}_p$	indicated Angle of Attack, formatted
γ	ratio of specific heats 1.4 for air
τ	time constant associated with the static pressure system
ΔP	difference between standard pressure and that existing at sea level

APPENDIX B

REAL VARIABLE TYPES

Variables	Units
Acceleration	ℓ/t^2
Angle	unitless
Angular Rate	$1/t$
Density	m/ℓ^3
Distance	ℓ
Mach	unitless
Pressure	F/ℓ^2
Speed or Velocity	ℓ/t
Time	t

APPENDIX C

Dependencies of Constants used in Physical and Empirical Equations

Suppose that the "true" physical equation,

$$y = f(x_1,\ldots,x_m;\ k_1,\ldots k_n) \qquad \text{for } l_i \le x_i \le u_i \ (i=1,\ldots,m) \tag{C-1}$$

where $x_1,\ldots,x_m$ are independent variables and $k_1,\ldots,k_n$ are physical constants (the actual values of which may have been empirically determined) is known. Then each k_i $(i=1,\ldots,n)$ can be described in terms of its particular physical dependencies.

On the other hand, suppose Equation (C-1) can be numerically represented, over some regions (possibly disjointed), by the set of p empirical equations,

$$Y_j = F_j\left(x_1,\ldots,x_m;\ K_{1j},\ldots,K_{N_j j}\right) \qquad \text{for } L_{ij} \le x_i \le U_{ij} \ (i=1,\ldots,m;\ j=1,\ldots,p) \tag{C-2}$$

where K_{ij} $\left(i=1,\ldots,N_j;\ j=1,\ldots,p\right)$ are empirical constants - i.e.,

$$
y = \begin{cases}
Y_1 \text{ for } L_{i1} \leq x_i \leq U_{i1} \\
Y_2 \text{ for } L_{i2} \leq x_i \leq U_{i2} \\
\quad \cdot \\
\quad \cdot \\
\quad \cdot \\
Y_r \text{ for } L_{ir} \leq x_i \leq U_{ir} \\
\quad \cdot \\
\quad \cdot \\
\quad \cdot \\
Y_p \text{ for } L_{ip} \leq x_i \leq U_{ip}
\end{cases} \qquad (i=1,\ldots,m). \tag{C-3}
$$

Then for any r^{th} set of constants K_{ir} $(i=1,\ldots,N_r)$, each and every one has the same dependencies as any other - viz., totally dependent on the process used to develop the equation

$$
Y_r = F_r\,(x_1,\ldots,x_m;\ K_{ir},\ldots,K_{n_r r}) \quad \text{for } L_{ir} \leq x_i \leq U_{ir}\ (i=1,\ldots,m). \tag{C-4}
$$

These dependencies would include characteristics of measuring devices and even the particular test data used to represent Equation (C-1).

(It is worth noting that if we consider Equation (C-2) for the special case of $p=1$ and

$$
\begin{aligned}
N_1 &= n \\
K_{i1} &= k_i\ (i=1,\ldots,n) \\
L_{i1} &= l_i\ (i=1,\ldots,m) \\
U_{i1} &= u_i\ (i=1,\ldots,m)
\end{aligned}
$$

that this resultant "empirical" equation becomes identical to Equation (C-1).)

APPENDIX D

NOTATION

While most readers are probably familiar with the mathematical notation used in this report, what follows is a list of notational symbols that, due to inconsistent usage in the literature, ought to be clarified here.

$x \equiv y$	x is identical to y
$x \approx y$	x is approximately equal to y
$\left(\frac{\partial}{\partial x_j} f(x_1,\ldots,x_n)\right)_{k_1,\ldots k_n}$	identical to $F(k_1,\ldots,k_n)$ for $F(x_1,\ldots,x_n) = \frac{\partial}{\partial x_j} f(x_1,\ldots,x_n)$
$\binom{n}{j}$	binomial coefficient $\frac{n!}{j!(n-j)!}$
$\sum_{R(j)} f(j)$	sum of all f(j) such that j is an an integer and relation R(j) is true.*
$O(f(x))$	any quantity F(x) such that $\lvert F(x)\rvert \leq N \lvert f(x)\rvert$ over the interval $\alpha \leq x \leq \beta$, where N is an unspecified constant.*

*For a detailed discussion, the reader is referred to Knuth, D.E., "The Art of Computer Programming," Vol. 1, 2nd ed., Addison-Wesley, Reading, Mass., 1973.

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— 8